Davis

Direction of a line

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REMOTE STORAGE

DIRECTION OF A LINE

J. B. DAVIS, ANN ARBOR.

For needle compass work the following is sufficient: In the figure, *B* is the Pole Star, α Ursae Minoris, mag. 2; *A* is δ Ursae Minoris, mag. 4.5, next to the Pole Star in the handle of the little dipper; *C* is 51 Cephei, mag. 5; *D* is 32 Camelopardalis, mag. 5.4.

The lines *A C* and *B D* intersect about 13 minutes of arc from the pole measured on a line *E F*, parallel to *A D*. The pole is toward the side *A B*.

Set a light over a stake. At some convenient distance south of the light fix the position of another stake by holding a plumb line in range with the light and the pole. A good way of doing this is to drive two tall stakes in the ground south from the lantern and nail a board horizontally to them so that a person seated can look under the board and view the stars *A*, *B*, *C*, *D*, Fig. 1. Hold the string of a plumb bob against the south side of the board while bringing it into range with the light and the pole. After the range is obtained mark the place of the string on the board. Repeat these trials, marking each position of the string. Place the marks on the board at different elevations to prevent one mark covering another. Select the average position of these marks and notch the board at this place. Transfer the position of the notch to the ground. Set a stake here. Drive a nail in the stake directly beneath the notch.

The two stars in the inner corner of the little dipper that are the more prominent, namely 19 and 21, or 19 and η, Ursae Minoris, are 25½ minutes of arc apart.

For something more precise one may use the process of the precise method given below and make the observations by means of a suspended plumb line, and a compass sight, or by the plumb line and peep sights cut in chips

19341

forced into a cleft stake and inclined to cover the star at its elongation. The practice can be readily adapted to the means at hand when once the method is understood, as given below.

Some knowlege of the names and relative positions of stars is needed. This is best obtained from one of the planispheres now published or from some good star map of the north polar region. Whitall's Planisphere, or Harrington's Planisphere, or R. A. Proctor's Star Atlas, are good for this purpose.

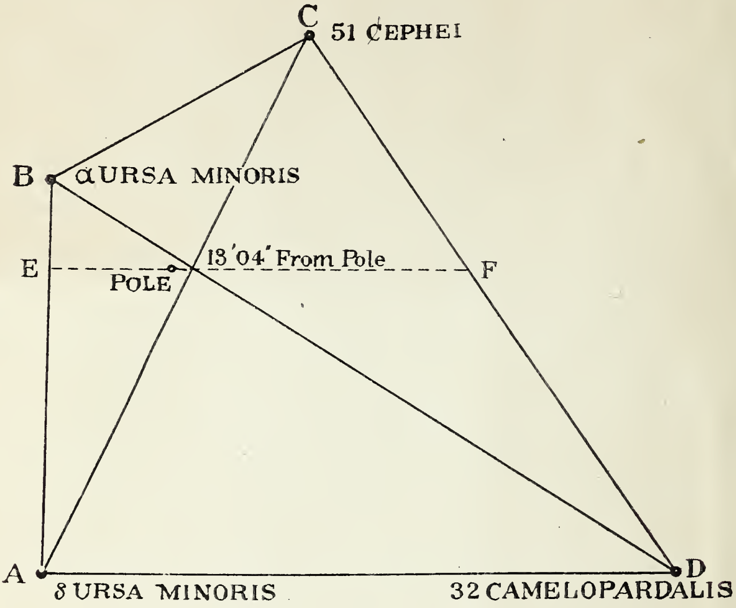

Fig 1.

There will also be needed a table of the Apparent Places of Fixed Stars, such as is given in the American Ephemeris or some similar work. This book also furnishes some other data that may be needed by a beginner in preparing for the observations. After some experience no such book will be needed before making the observations (only afterwards for the reductions), so one can observe at once without any previous calculations.

Observe the following conditions and notation. It is presumed that the observer has a knowledge of the elementary astronomical terms in common use, the kinds of time, and the use of the ordinary mathematical tables, such as Hussey's, Jones,' Searles,' or Vega.

ϕ = latitude of place of observation.

δ = declination, δ_0, δ_1, δ_2, etc., for particular stars, and δ_m the least declination that can be used.

a = right ascension, a_0, a_1, a_2, etc., for particular stars, a_m the least and a_n the greatest value of a that can be used.

t = hour angle with same subscripts as a,—t_n=6 hours.

h = altitude with similar subscripts.

T = astronomical time of any observation, that is, its time from noon, with similar subscripts.

ν_0 = "Siderial time of Mean Noon" at Washington for the date of the observation, taken from the table headed, "Solar Ephemeris" in the American Ephemeris. In case some other ephemeris is used, as that for Greenwich, or a Nautical Almanac, get from it the sidereal time of mean noon, or the right ascension of the mean sun, for the date of the observation. This will be ν_0.

A = azimuth of a star at its elongation, with subscripts similar to those for δ.

The limiting values of δ, t, and a depend upon the limit of altitude which the instrument to be used is capable of observing. Examine the instrument to be used and ascertain the greatest altitude it will give comfortably. Try it with the telescope both sides up. In common field transits this is 45° to 55°, after which the eye-piece comes in the way of the edge of the plate. This is h_n.

Then
$$\sin \delta_m = \frac{\sin \phi}{\sin h_n}$$

and
$$\cos t_m = \frac{\tan \phi}{\tan \delta_m}$$

If ϕ is not known take it from a map or guess at it, being sure to use a value a little too large.

By judgement assume T_m the earliest hour from noon when observations can begin on account of the daylight, and T_n the latest hour from noon at which it is desired to continue the observations.

Then for Eastern Elongations,

$$\begin{aligned} a_m &= T_m + \nu_0 + t_m \\ a_n &= T_n + \nu_0 + 6^h \\ a_m &= T_m + \nu_0 - 6^h \\ a_n &= T_n + \nu_0 - t_m \end{aligned}$$

If α_m or α_n is more than 24^h, reject 24^h.

From the tables of Apparent Places of Fixed Stars choose the stars of proper magnitude whose declinations are greater than δ_m and whose right ascensions are between α_m and α_n for the eastern and western elongations, as the case may be. Find the times of the elongations of these stars, as follows:

Find the hour angle by

$$\cos t = \frac{\tan \phi}{\tan \delta}$$

and the times by $T = \alpha \pm t - \nu_0$. $+\,t$ for western and $-\,t$ for eastern elongations. If α is not large enough to subtract t and ν_0 from, or if $\alpha + t$ is less than ν_0, add 24 hours. Having found the times, revise the list, setting down the times in the order of their succession with the names of the proper stars against them, their magnitudes, and a note of their elongations, E. or W. It may be well to add their declinations also to save looking them up again.

In a certain set of observations for March 10th, 1875, the instrument to be used was found to indicate an altitude of 50° with comfort. The latitude was thought to be something less than $43° = \phi$.

$$\begin{array}{lll} \sin \delta_m = \dfrac{\sin 43°}{\sin 50°} & \log \sin 43° & 9.8338 \\ & \text{“} \quad \text{“} \quad 50° & \underline{9.8843} \\ \delta_m = 62°\ 55' & \text{“} \quad \text{“} \quad \delta_m & 9.9495 \end{array}$$

$$\begin{array}{lll} \cos t_m = \dfrac{\tan \phi}{\tan \delta_m} = \dfrac{\tan 43°}{\tan 62°\ 55'} & \log \tan \quad 43° & 9.9697 \\ & \text{“} \quad \text{“} \quad 62°\ 55' & \underline{0.2913} \\ t_m = 61°\ 31' = 4^h\ 6^m & \text{“} \quad \cos t_m & 9.6784 \end{array}$$

The earliest hour work could begin was 7.30 P. M. Therefore $T_m = 7^h\ 30^m$. The latest hour it was desired to observe 11^h — T_n. ν_0, from the Solar Ephemeris Table for 1875–3–10 is about $23^h\ 12^m$.

E. Elong. $\alpha_m = 7^h\ 30^m + 23^h\ 12^m + 4^h\ 6^m = 34^h\ 48^m$ or $10^h\ 48^m$
$\alpha_n = 11^h + 23^h\ 12^m + 6^h = 40^h\ 12^m$ or $16^h\ 12^m$
W. Elong. $\alpha_m = 7^h\ 30^m + 23^h\ 12^m - 6^h = 24^h\ 42^m$ or $0^h\ 42^m$
$\alpha_n = 11^h + 23^h\ 12^m - 4^h\ 6^m = 30^h\ 06^m$ or $6^h\ 06^m$

The following list of stars was selected from the table in the American Ephemeris giving the Apparent Places of Fixed Stars. The declinations are greater than 62° 55′ and the magnitudes are found from the preceding table of Mean Places.

E. Elong.

Name	α	δ	Mag.	T, about
λ Draconis,	$11^h\ 24^m$	70° 01′ 09.″4	3.4	$7^h\ 31^m$
4 Draconis,	$12^h\ 06^m$	78° 18′ 30 ″1	5.4	$7^h\ 39^m$
κ Draconis,	$12^h\ 28^m$	70° 28′ 26.″4	3.4	$8^h\ 34^m$
32 Camelop,	$12^h\ 48^m$	84° 05′ 19.″3	5.4	$7^h\ 58^m$
α Draconis,	$14^h\ 01^m$	64° 58′ 02.″9	3.4	$10^h\ 32^m$
etc.	etc.	etc.	etc.	etc.

W. Elong.

Name	α	δ	Mag.	T, about
α Ursæ Minoris,	$1^h\ 11^m$	88° 37′ 47.″4	2.0	$7^h\ 55^m$
50 Cassiopeæ,	$1^h\ 53^m$	71° 49′ 07.″5	4.0	$7^h\ 30^m$
ι Cassiopeæ,	$2^h\ 19^m$	66° 50′ 34.″1	4.0	$7^h\ 33^m$
etc.	etc.	etc.	etc.	etc.

the times, T, being found by the equation as follows:

$\cos. t = \dfrac{\tan \phi}{\tan \delta}$ For λ Draconis $\delta = 70°\ 01'$ $\alpha = 11^h\ 24^m$
$\phi = 43°$ $\nu_0 = 23^h\ 12^m$

log tan 43° 9.9697
" " 70° 01′ 0.4393
" cos t 9.5304 $t = 70°\ 01' = 4^h\ 41^m$
$23^h\ 12^m$
$27^h\ 53^m$
$24^h + 11^h\ 24^m = 35^h\ 24^m$
$T = 7^h\ 31^m$ E. Elong.

For α Ursæ Minoris, $\delta = 88°\ 38'$ $\alpha = 1^h\ 11^m$

$$
\begin{array}{llrr}
\text{log tan } 43^\circ & 9.9697 & & \\
\text{`` \ `` } 88^\circ\ 38' & \underline{1.6224} & & 1^h\ 11^m \\
\text{`` cos } t & 8.3473\ t = 88^\circ\ 43' & = & 5^h\ 56^m \\
 & & & \underline{24^h\ 00^m} \\
 & & & 31^h\ 07^m \\
 & & & \underline{23^h\ 12^m} \\
 & & T = & 7^h\ 55^m \quad \text{W. Elong.}
\end{array}
$$

The revised list was then made out as follows:

T	Elong.	Mag.	Name	δ
7.30	W.	4.0	50 Cass.	71° 49′ 07.″5
7.31	E.	3.4	λ Draco.	70° 01′ 09.″4
7.33	W.	4.0	ι Cass.	66° 50′ 34.″1
7.39	E.	5.4	4 Draco.	78° 18′ 30.″1
etc.	etc.	etc.	etc.	etc.

It may be noticed that h_n must be considerably larger than ϕ in order for the instrument to be used at all. The work of preparing such a list can readily be abbreviated. With some experience a list may be written at once without figuring. When continually at work at the same place the list may be extended from day to day by simple inspection. The changes in the values of α_m and α_n depend upon the change in value of ν_0. The change in T is mainly due to the change in ν_0. The extreme detail in the above work is for a novice,—not as an example of practice. Western elongations are to be preferred for the smaller field instruments, because, if the star can be observed before it reaches its elongation it can be seen at its elongation. The data used being approximate, it may happen that a star at its eastern elongation, may pass a little too high for the instrument.

Provide some means of illuminating the cross wires so they can be seen at the same time the star is. Instruments of the more precise grades intended for night work, and mining instruments, are fitted with apparatus for this purpose. The common engineers transit for field use generally is not. For such an instrument tie a piece of unglazed white paper (common newspaper) over the objective by a string around the telescope tube back of the lens, with a hole in the paper centrally over the objective. This hole may be from ¼″ to ½″, according to the size of the lens and its aperture. Light shining upon the outside of this paper will render the cross wires visible. A lantern with a lens throwing light in one direction is best. The common lantern may be made use of by nearly surrounding it with something to shut off the light. The light must not shine in the observer's face. It must therefore shine obliquely from the side upon the paper over the objective. It is possible to observe without an assistant, but difficult. The paper over the objective is a protection to it. The light may be toned to any faintness within the telescope by withdrawing the lantern or changing the obliquity of the the beam of light. The lantern may also be used to read the plate. It must not be held too near the plates or its heat may change the readings.

Choose some point of the line whose direction is sought, or reference line, from which a clear view of the stars to be observed may be obtained, for an instrument point. Fasten a lantern at some other point of the reference line to sight to. A common lantern about a mile away will do very well. If necessary to have it nearer, a fainter light may be made by covering part of the lantern globe. This light should be faint enough to look about like a faint star, so that no change of focus of the objective will be required.

In good season for beginning the observation, set up the instrument over its point of the reference line. Sight to the distant lantern on its point of the reference line. In time for the first star, illuminate the cross wires and set the line of sight upon the distant lantern. See that the spindle clamp is secure. Read the plate. Record the readings. Release the plate clamp. Focus the objective for the stars, if it needs it, for which purpose the moon, or a planet, is useful. Point the telescope at the star to be observed and bring it into its field of view. Illuminate the cross wires. Set the vertical wire, by means of the plate clamp and slow motion screw, on the star. Keep this wire on the star very carefully as it approaches its elongation, which may be known by the increased time taken by the star to cross the wire. Follow the star with the plate slow motion screw, keeping the vertical wire exactly on it till it appears to pass directly along the wire. Read the plate. Record the readings marked for the star observed. Point the telescope to the distant lantern. Illuminate the cross wires and set the line of sight again upon this lantern by means of the plate clamp and slow motion screw. Read the plate. Record the readings. Compare them with those before observing the star. If they differ too much from them the observation must be rejected. The amount of this difference depends upon the instrument and the class of the work.

In time for another star, read the plate with the line of sight set again upon the reference line and record the readings. Release the plate clamp, point the telescope for the star, illuminate the cross wires, and observe it as the first one was observed. Record the plate readings for this star. Set the line of sight again upon the reference line by means of the plate clamp and slow motion screw. Read the plate. Record the readings, compare them with those before observing the star, and thus examine the position of the instrument as to stability. So do for every star observed.

Observe at least three stars in this way that give good observations, and as many more as may be desirable or possible.

The observations are reduced as follows: Find, from the plate readings on the reference line and those for each star, the angles between the vertical circle of the reference line and those of the stars. From these angles find half the angle between the vertical circles of a pair of the stars. There are two cases: when the vertical circle of the reference line is between the stars, and when it is not. In the first instance, take the half sum of the horizontal angles, in the second, their half difference. If the stars were at the same elongation, that is, if both *E*, or both *W*, this will be ½ (A_0-A_1) in the equations below, if one is *E* and one *W*, this will ½ (A_0+A_1). Take from the Ephemeris the apparent declinations, δ_0 and δ_1, of these stars for the date of the observation. By means of the following equations, using the proper one, compute ½ (A_0+A_1) or ½ (A_0-A_1), as the case may be:

$$\text{Tan } \tfrac{1}{2}(A_0+A_1) = -\tan \tfrac{1}{2}(A_0-A_1) \cot \tfrac{1}{2}(\delta_0+\delta_1) \cot \tfrac{1}{2}(\delta_0-\delta_1).$$
$$\text{Tan } \tfrac{1}{2}(A_0-A_1) = -\tan \tfrac{1}{2}(A_0+A_1) \tan \tfrac{1}{2}(\delta_0+\delta_1) \tan \tfrac{1}{2}(\delta_0-\delta_1).$$

Having thus obtained ½ (A_0+A_1) and ½ (A_0-A_1), find A_0 and A_1. These are the angles from the meridian to the vertical circles of the stars. Apply these values of A_0 and A_1 to the horizontal angles from the vertical circle of the reference line to the vertical circles of the stars, first above directed to be found, to get the horizontal angle from the meridian to the vertical circle of the reference line, or its direction. Notice how these values of A_0 and A_1 must be applied in order to give the quantity sought. The two values found for the direction of the line should be identical out to the last figure. If not it shows an error in figuring which must be found and corrected.

Three stars must be observed in order to get a check upon both the figures and the work. In combining the observations as above directed, those upon two stars at the same elongation *having nearly the same* δ must not be put in the same pair, because cot ½ $(\delta_0-\delta_1)$ will be so large a multiplier for the errors of observation that the result will be found worthless. By this method the observations may have any interval of time between them. When reducing, use the apparent declinations of the stars at the time when they were observed. This fact permits all observations from the same reference line to be combined. This makes the final result more trustworthy. The observations are combined in pairs. This gives the greatest number of results from a given list of observations.

The latitude of the place of observation enters into the work only in such a loose manner as to enable one to use this method where the latitude is not known. After some experience it will not be needed at all. The observations may be made at any time and reduced at any other.

After the azimuths, A_0, A_1, etc., of the stars are found, the latitude may be found by:

$$\cos\phi = \frac{\cos\delta_0}{\sin A_0} = \frac{\cos\delta_1}{\sin A_1} = \frac{\cos\delta_2}{\sin A_2} = \text{etc.}$$

This shows that the latitude may be found by the measurement of horizontal angles alone.

Of the above list of stars the following notes of observations upon three of them are taken as an example:

		Ver. A.	Ver. B.
Readings on Ref. Line,		15° 14′ 10″	195° 14″ 20″
" " α Dr.,	E. Elong.,	132° 39′ 50″	312° 40′ 00″
" " α Urs. Min.,	W. "	103° 18′ 40″	283° 18′ 40″
" " κ Dr.,	E. "	132° 01′ 00″	312° 01′ 00″

The work of reduction is as follows:

Angle from Ref. Line to λ Dr.,		117° 25′ 40″	117° 25′ 40″
" " " " " α Ur. Min.,		88° 04′ 30″	88° 04′ 20″
" " " " " κ Dr.,		116° 46′ 50″	116° 46′ 50″
	117° 25′ 40″	117° 25′ 40″	116° 46′ 45″
	88° 04′ 25″	116° 46′ 45″	88° 04′ 25″
2.)	29° 21′ 15″	0° 38′ 55″	28° 42′ 20″
	14° 40′ 37″.5	0° 19′ 27″.5	14° 21′ 10″
	½ $(A_0 + A_1)$	½ $(A_0 - A_2)$	½ $(A_1 + A_2)$
δ = Declinations	70° 01′ 09″.4	70° 01′ 09″.4	88° 37′ 47″.4
δ =	88° 37′ 47″.4	70° 28′ 26″.4	70° 28′ 26″.4
2.)	158° 38′ 56″.8	140° 29′ 35″.8	159° 06′ 13″.8
Half sum of dec's,	79° 19′ 28″.4	70° 14′ 47″.9	79° 33′ 06″.9
2.)	—18° 36′ 38″.0	—0° 27′ 17″.0	18° 09′ 21″.0
Half diff. of dec's,	— 9° 18′ 19″.0	—0° 13′ 38″.5	9° 04′ 40″.5

Tan ½ $(\delta_0 + \delta_1)$	0.7246723	cot ½ $(\delta_0 + \delta_2)$	9.5552189
" ½ $(\delta_0 - \delta_1)$	9.2144487	" ½ $(\delta_0 - \delta_2)$	2.4014043
" ½ $(A_0 + A_1)$	9.4182506	tan ½ $(A_0 - A_2)$	7.7528364
" ½ $(A_0 - A_1)$	9.3573916	" ½ $(A_0 + A_2)$	9.7094596

½ $(A_0 - A_1)$	12° 49′ 40″	½ $(A_0 + A_2)$	27° 07′ 21″.4
½ $(A_0 + A_1)$	14° 40′ 37″.5	½ $(A_0 - A_2)$	0° 19′ 27″.5
A_0	27° 30′ 17″.5	A_0	27° 26′ 48″.9
A_1	1° 50′ 57″.5	A_2	26° 47′ 53″.9

tan $\frac{1}{2}(\delta_1 + \delta_2)$ 0.7342345
" $\frac{1}{2}(\delta_1 - \delta_2)$ 9.2035192
" $\frac{1}{2}(A_1 + A_2)$ 9.4080330
—tan $\frac{1}{2}(A_1 - A_2)$ 9.3457867

$\frac{1}{2}(A_1 - A_2)$ — 12° 30′ 03″.1
$\frac{1}{2}(A_1 + A_2)$ 14° 21′ 10″
A_1 1° 51′ 06″.9
A_2 26° 51′ 13″.1

Ang. to λ Dr. 117° 25′ 40″
A_0 27° 30′ 17″.5
Az. of Line, 89° 55′ 22″.5

Ang, to κ Dr. 116° 46′ 45″
A_2 26° 47′ 53″.9
89° 58′ 51″.1

Ang. to α Ur. Min, 88° 04′ 25″
A_1 1° 51′ 06″.9
Az. of Line, 89° 55′ 31″.9

Ang. to α Ur. Min. 88° 04′ 25″
A_1 1° 50′ 57″.5
Az. of Line, 89° 55′ 22″.5

Ang. to λ Dr. 117° 25′ 40″
A_0 27° 26′ 48″.9
89° 58′ 51″.1

Ang. to κ Dr. 116° 46′ 45″
A_2 26° 51′ 13″.1
Az. of Line, 88° 55′ 31″.9

Reject the middle value of the azimuth of the line and the

Final Azimuth of Line = 89° 55′ 26″.7.

As the plate was numbered from 0° to 360° in right handed rotation and all angles read from the reference line round to the right, the result is the angle from the reference line reckoned to the right to the meridian. Since the instrument looked to the northern portion of the heavens this angle reads to the north point of the horizon and the direction of this line is

N. 89° 55′ 26″.7 W.

The middle value of the azimuth of the line was rejected because of its manifest error, being some 3′ 25″ different from the others, which would exclude it from 10″ observations. It is thus in error because it was obtained from two stars at the same elongation whose declinations were nearly the same. The calculations were given to show the worthlessness of such a combination. A more trustworthy value of this angle from other sources is 89° 55′ 30″. By this one may perceive the value of the middle result. While some discrepancy appears in the original readings the final result is quite good. The changes of temperature the parts of the instruments experience in night work out of doors with lanterns held close to them works mischief with the readings, and calls for constant back readings on the reference line.

To get the latitude,

$$\begin{array}{ll@{\qquad\qquad}ll} \cos\delta_0 & 9.5336500 & \cos\delta_2 & 9.5240514 \\ \sin A_0 & 9.6644764 & \sin A_2 & 9.6548625 \\ \cline{1-2}\cline{3-4} \cos\phi & 9.8691736 & \cos\phi & 9.8691889 \\ \phi & 42^\circ\ 16'\ 37'' & \phi & 42^\circ\ 16'\ 29''.3 \end{array}$$

$$\phi = 42^\circ\ 16'\ 33''.1$$

More trustworthy value from other sources,

$$\phi = 42^\circ\ 16'\ 30''.$$

By this may be seen the value of the method. Had the observations and reductions been extended somewhat, no doubt very close results could have been obtained.

It may happen that one may be provided with a comparatively poor instrument yet need to determine the direction of a line with some precision. Put the instrument in as good working condition as possible and in as good adjustment as it can be. Observe a star with it as before directed but use no reference line and read no angles. After pointing to the star, point downwards and set a stake marked for the star with a point carefully marked on it in line. Do the same for two or more stars, setting the stakes as far away as convenient yet near enough for the points on them to be well located. Reverse the telescope about both axes and level up the instrument anew after observing each star. Make notes showing the position of the telescope when pointing to each star. In combining the observations in pairs, place together two stars for which the notes show the telescope to have been in its two different positions. By so doing the imperfections of the instrument will do less harm.

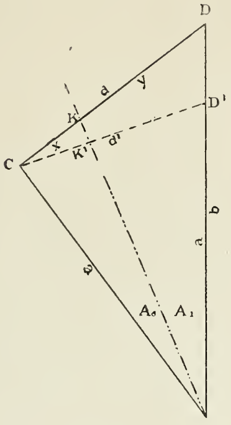

Fig 2.

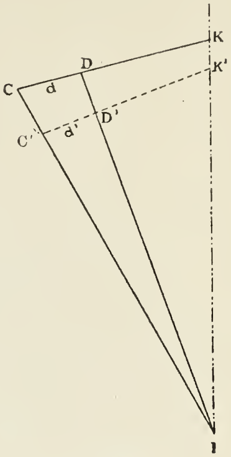

Fig 3.

If I, in Fig. 2, shows the place of the instrument, C and D, the stakes for two stars, one at its E. elongation and the other at its W. elongation, and $I\ K$ the meridian, make the following measurements carefully with a steel tape or similar tool,—$C\ D = d$; $I\ C = a$; and $I\ D = b$.

Then $x - y = d\dfrac{a \cos \delta_0 - b \cos \delta_1}{a \cos \delta_0 + b \cos \delta_1}$, where $x = C\ K$

and $y = K\ D$, the segments of $C\ D$ formed by K, the point where the meridian from I intersects $C\ D$. $x + y = d$. Find x and y and locate the point K with the instrument and steel tape. As a check on the figuring use

$$\frac{x}{y} = \frac{a \cos \delta_0}{b \cos \delta_1}$$

Since $A_0 = C\ I\ K$ and $A_1 = K\ I\ D$, $C\ I\ D = A_0 + A_1$.

Then Tan $\frac{1}{2}(A_0 - A_1) = -\tan \frac{1}{2}(A_0 + A_1) \tan \frac{1}{2}(\delta_0 + \delta_1)$ tan $\frac{1}{2}(\delta_0 - \delta_1)$ as before when A_0 and A_1 become known and

$$\cos \phi = \frac{\cos \delta_0}{\sin A_0} = \frac{\cos \delta_1}{\sin A_1} \text{ as before.}$$

Whence it appears that one may obtain the direction of the meridian and the latitude of the place without reading angles at all. A_0 and A_1 may be found directly from the triangles $C\,I\,K$ and $K\,I\,D$.

When the stars are at the same elongation, as in Fig. 3,

$$d = x - y, \text{ and}$$
$$x + y = d\,\frac{a \cos \delta_0 + b \cos \delta_1}{a \cos \delta_0 - b \cos \delta_1}\,.$$

Should D' be set in line from I to D so that $I\,D' = I\,C = a$, as in Fig. 2, the figuring would be simpler. Measure $C\,D' = d'$, and

$$x' - y' = d' \tan \tfrac{1}{2}\,(\delta_0 + \delta_1) \tan \tfrac{1}{2}\,(\delta_0 - \delta_1)$$

In Fig. 3, where the stars are at the same elongation, C' and D' may be set in the lines $I\,C$ and $I\,D$, respectively, and at the same distance (some simple number of feet) from I. Measure

$$C'\,D' = d'. \quad x' = C'\,K' \text{ and } y' = K'\,D'.$$
$$d' = x' - y',$$
and
$$x' + y' = -d' \cot \tfrac{1}{2}\,(\delta_0 + \delta_1) \cot \tfrac{1}{2}\,(\delta_0 - \delta_1).$$

Here the same caution must be used about combining values of δ that are too near alike, as is noted in reducing observations in which angles are read.

Of course C' and D' can be set at the same convenient distance from I in Fig. 2, and the work proceed as outlined. C' and D' might, as well, be set beyond C and D.

For still more precise processes (though not more precise methods) see Handbook of Practical Astronomy by W. W. Campbell of the Lick Observatory, Chapter X, and elsewhere in the book, also Field Astronomy for Engineers, by Geo. C. Comstock of the Washburn Observatory, Wisconsin.